# Statistics

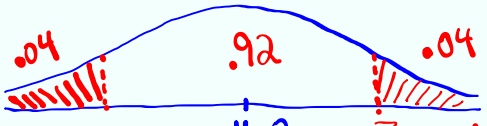
# Lecture 10

Feb 19-8:47 AM

Class QZ 7 Open notes:

Drawing, shading, labeling, and full TI command required.

1) Find $z_{\alpha/2}$ for $\alpha = .08$.




$z_{.04} = \text{invNorm}(.96, 0, 1)$

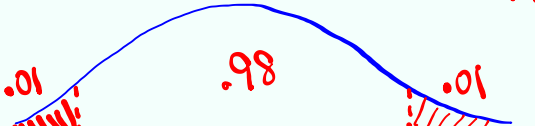
$\approx \boxed{1.751}$

2) Find $t_{\alpha/2}$ for 98% conf. interval with $df = 49$.

Middle Area .98

$1 - \alpha = .98$

$\alpha = .02$, $\alpha/2 = .01$




$t_{.01} = \text{invT}(.99, 49)$

$\approx \boxed{2.405}$

May 1-7:42 AM

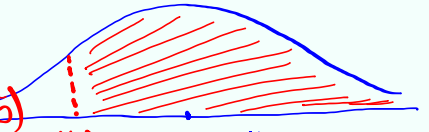
Class QZ 6 open notes:

Given $N(125, 15)$, $n=4$

1) Find $P(\bar{x} > 110)$

= normalcdf(110, E99, 125, 7.5)

$\approx$ .977 $\approx$ 97.7% $\approx$ 98%



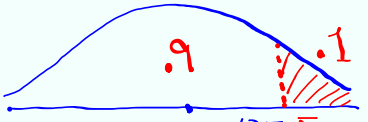

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 125 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{4}} = 7.5 \end{cases}$

2) Find $\bar{x} = P_{90}$, Round to whole #.

$\bar{x}$ = invNorm(.9, 125, 7.5)

= 134.612

$\approx$ 135




CLT $\begin{cases} \mu_{\bar{x}} = 125 \\ \sigma_{\bar{x}} = 7.5 \end{cases}$ $\bar{x}$

Apr 24-10:50 AM

Estimating Parameters:

SG 21 & SG 22

we use Samples to estimate population

Sample $\longleftrightarrow$ Statistic  Population $\longleftrightarrow$ Parameter

we use statistic to estimate Parameter.

| To estimate | we use |
|---|---|
| Population Proportion $p$ | Sample Proportion $\hat{p}$ |
| Population Mean $\mu$ | Sample Mean $\bar{x}$ |
| Population Standard deviation $\sigma$ | Sample Standard deviation $s$ |

Point-estimate

Best Guess

May 1-8:22 AM

Estimation of a parameter is a range of value which is called Confidence interval.

Every Confidence interval comes with Confidence level $(1-\alpha)\cdot 100\%$.

where $0<\alpha<1$, $\alpha/2$ is the area on each tail and $1-\alpha$ is the middle area on the distribution graph.

If $\alpha$ not given → use .05

If C-level not given → use 95%.

May 1-8:29 AM

Estimating Population Proportion P:

$$<P<$$

$$\hat{P}-E<P<\hat{P}+E$$

Sample Proportion
Point-estimate

Margin of error

$\hat{P}=\frac{x}{n}$ ← # of favorable responses
← Sample Size

$\hat{q}=1-\hat{P}$

$E=Z_{\alpha/2}\cdot\sqrt{\frac{\hat{P}\hat{q}}{n}}$

Critical value for $(1-\alpha)\cdot 100\%$ C-level.

May 1-8:34 AM

I Surveyed 100 Students and 80 of them had iPhone. $n=100$ $x=80$

$\hat{p}=\frac{x}{n}=\frac{80}{100}=.8$

$\hat{q}=1-\hat{p}=1-.8=.2$

Find Conf. interval for the population of all Students that have iPhone using 90% C-level.

$\hat{p}-E<p<\hat{p}+E$

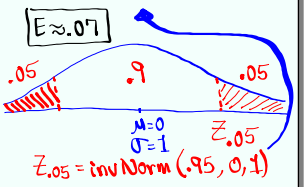
$E=z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$.8-.07<p<.8+.07$

$=1.645\sqrt{\frac{(.8)(.2)}{100}}$

$.73<p<.87$

$E\approx.07$




we are 90% Confident that between 73% & 87% of all Students have iPhone.

$z_{.05}=$ invNorm $(.95,0,1)$

using TI:

STAT → TESTS ↓ 1-PropZInt

$x=80$

$n=100$

C-level: .9

Calculate

$.73<p<.87$

$E=\frac{.87-.73}{2}=\frac{.14}{2}=.07$

$\hat{p}=\frac{.87+.73}{2}=\frac{1.6}{2}=.8$

May 1-8:41 AM

I Surveyed 240 students and 96 of them were fan of online classes. $n=240$ $x=96$

C-level: .99

Find 99% Confidence interval for the proportion of all Students that are fan of online classes.

STAT → TESTS ↓ 1-PropZInt

$.32<p<.48$

$x=96$

$n=240$

C-level: .99

Calculate

we are 99% Confident that between 32% and 48% of all Students are fan of online classes.

$E=\frac{.48-.32}{2}=\frac{.16}{2}=.08$

$\hat{p}=\frac{.48+.32}{2}=\frac{.8}{2}=.4$

May 1-8:55 AM

I surveyed 125 students and 8.2% of them were sadly Smokers.

$n = 125$ $\quad \hat{p} = .082$

$x = n\hat{p} = 125(.082) = 10.25 \approx \boxed{11}$

if decimal → Round-up

No C-level → use .95

Find Confidence interval for the proportion of all Students that are Smokers.

1-PropZInt

$x = 11$

$n = 125$

C-level: .95

$\boxed{.04 < p < .14}$

We are 95% Confident that between 4% & 14% of all Students are Smokers.

$E = \frac{.14 - .04}{2} = \boxed{.05}$ $\qquad \hat{p} = \frac{.14 + .04}{2} = \boxed{.09}$

May 1-9:04 AM

Estimating Population mean $\mu$:

$< \mu <$

$\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample mean, Point-estimate

$E$: Margin of error

Case I: $\sigma$ Known

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

STAT → TESTS ↓ ZInterval

inpt: Stats

⋮

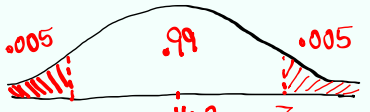
Calculate

May 1-9:26 AM

Given: $\bar{x}=86$ $n=32$ $\sigma=12$ C-level: 99%

Find C.I. for pop. mean.

$\sigma$ Known → $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 2.576 \cdot \frac{12}{\sqrt{32}} \approx 5.465 \approx 5$




$z_{.005} = \text{invNorm}(.995, 0, 1) \approx 2.576$

$\bar{x} - E < \mu < \bar{x} + E$

STAT → TESTS ↓ ZInterval

$86 - 5 < \mu < 86 + 5$

$81 < \mu < 91$

Inpt: Stats

$\sigma = 12$

$\bar{x} = 86$

$n = 32$

C-level: .99

Calculate

$E = \frac{91-81}{2} = 5$

$\bar{x} = \frac{91+81}{2} = 86$

$81 < \mu < 91$

Since $\bar{x}$ is a whole #

May 1-9:31 AM

I surveyed 36 students, their mean age was 32.5 yrs. $n=36$ $\bar{x}=32.5$

It is known that standard deviation of ages of all students is 8.2 yrs. $\sigma = 8.2$

No C-level → .95

Find Conf. interval for the mean age of all students.

$29.8 < \mu < 35.2$

$\sigma$ Known → ZInterval

Inpt: Stats

$\sigma = 8.2$

$\bar{x} = 32.5$ ← 1-Dec.

$n = 36$

C-level: .95

Calculate

$E = \frac{35.2 - 29.8}{2} = 2.7$

$\bar{x} = \frac{35.2 + 29.8}{2} = 32.5$

May 1-9:40 AM

Estimating Population mean $\mu$:

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x}$: Sample mean, Point-estimate

$E$: Margin of error

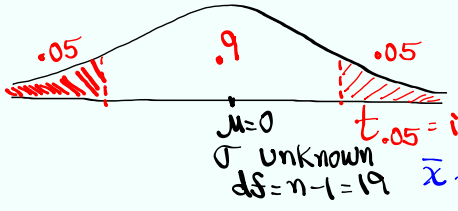
| Case I: $\sigma$ Known | Case II: $\sigma$ Unknown |
|---|---|
| $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ | $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$, $df = n-1$ |
| STAT → TESTS ↓ ZInterval | STAT → TESTS ↓ TInterval |
| inpt: Stats | inpt: Stats |
| ⋮ | ⋮ |
| Calculate | Calculate |

May 1-9:26 AM

Given: $\bar{x} = 84$, $S = 10$, $n = 20$, C-level = 90%

Find Conf. interval for population mean.

$\sigma$ unknown $\rightarrow E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 1.729 \cdot \frac{10}{\sqrt{20}} \approx 3.866 \approx 4$




$\bar{x} - E < \mu < \bar{x} + E$

$84 - 4 < \mu < 84 + 4$

$80 < \mu < 88$

STAT → TESTS ↓ TInterval

inpt: STATS

$\bar{x} = 84$

$S = 10$

$n = 20$

C-level: .9

$E = \frac{88 - 80}{2} = 4$

$\bar{x} = \frac{88 + 80}{2} = 84$

$80 < \mu < 88$

whole #

May 1-9:54 AM

12 randomly selected nurses had a mean monthly salary of \$6400 with standard deviation of \$500.

$n = 12$ $\bar{x} = 6400$ $S = 500$ $\rightarrow df = 12-1 = 11$

no C-level: .95

Find Conf. interval for the mean salary of all nurses.

$\sigma$ unknown $\rightarrow$ TInterval

Since $\bar{x}$ is a whole #

$6082 < \mu < 6718$

$E = \frac{-}{2} = 318$

$\bar{x} = \frac{+}{2} = 6400$

May 1-10:03 AM

I randomly selected 10 exams. Here are the Scores:

| | | | | |
|---|---|---|---|---|
| 84 | 72 | 93 | 100 | 65 |
| 78 | 80 | 90 | 95 | 85 |

Find

1) $\bar{x} = 84.2 \approx 84$
2) $S = 10.809 \approx 11$
3) $n = 10$

} whole #

4) Find 99% Conf. interval for the mean of all exams.

$\sigma$ unknown $\rightarrow$ TInterval

Since $\bar{x}$ is a whole #

$73 < \mu < 95$

$E = \frac{95-73}{2} = 11$

$\bar{x} = \frac{95+73}{2} = 84$

May 1-10:12 AM

How to determine minimum Sample Size: $n$

1) Population Proportion

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$ → with Some algebra work → isolate $n$

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

if decimal → Round-up

if $\hat{p}$ & $\hat{q}$ are unknown, use .5 for each

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

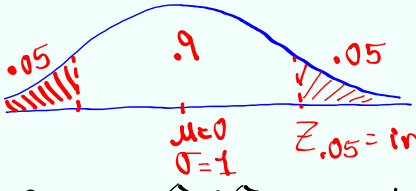
May 1-10:30 AM

Find minimum Sample Size needed to Construct 90% Conf. interval for population Proportion with $\hat{p} = .35$ and $E = .05$.

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.35)(.65)\left(\frac{1.645}{.05}\right)^2$$

$= 246.24\ldots$

247




Suppose $\hat{p}$ & $\hat{q}$ are unknown and $E = .04$

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.645}{.04}\right)^2 = 422.816\ldots$$

423

May 1-10:35 AM

## 2) Population mean

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$ → with some algebra work

Solve for $n$

$$n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

if decimal → Round-up

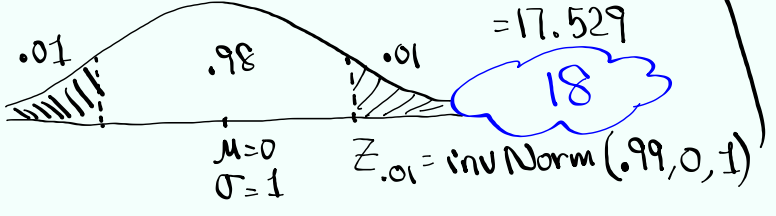
If $\sigma$ is unknown →

$$n = \left( \frac{z_{\alpha/2} \cdot s}{E} \right)^2$$

May 1-10:43 AM

Find min. Sample Size needed to Construct 98% Conf. interval for pop. mean with $\sigma = 18$ and $E = 10$.

$$n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left( \frac{2.326 \cdot 18}{10} \right)^2 = 17.529$$




18

Redo if $E = 5$.

$$n = \left( \frac{2.326 \cdot 18}{5} \right)^2 = 70.117 \approx 71$$

May 1-10:46 AM